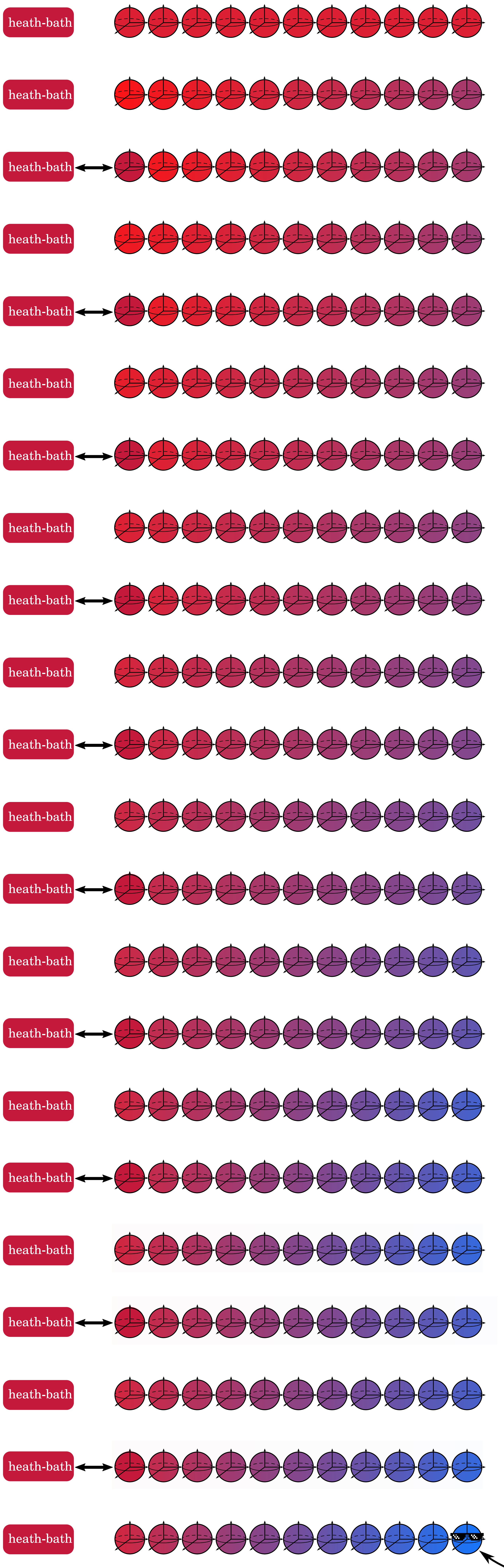
Oblivious algorithmic cooling

Sadegh Raeisi[1], Maria Kieferova[2], Michele Mosca[3-6]
[1] Department of Physics, Sharif University of Technology, Tehran 1458889694, Iran
[2] UTS, New South Wales 2109, Australia
[3] IQC, University of Waterloo, Ontario N2L 3G1, Canada 3
[4] Department of Combinatorics and Optimization, University of Waterloo, Ontario N2L 3G1, Canada
[5] Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2L 2Y5, Canada
[6] Canadian Institute for Advanced Research, Toronto, Ontario M5G 1Z8, Canada



the cool qubit

Heat-bath algorithmic cooling (HBAC)

Algorithmic cooling refers to protocols for extracting a very pure qubit out of an ensemble of qubits in thermal states [1]. The first algorithmic cooling techniques consisted only of unitary operations on the qubits which made them very limited. Lower temperatures (higher purity), can be achieved using heat-bath algorithmic cooling (HBAC) methods [2], which assume that parts of the cooled system can exchange heat with a heat bath. We assume that the heat bath can only reset the qubits to its temperature, although more complex thermalization strategies have been recently proposed [3]. Interestingly, even with the help of a heat bath, there is a lower limit on the achievable temperature. This limit is achieved by the partner-paring algorithm (PPA) [2,4].

The problem

One of the main challenges with HBAC techniques (including PPA) is that they are highly sensitive to the state of the cooled qubits. That is, the operations in each iteration depend on the state of the qubits. This means that picking the right operation requires nearly-perfect knowledge about the system. In this work, we give an algorithm for reaching the asymptotic cooling state with a fixed, state-independent operation in each iteration. In the language of quantum thermodynamics, this translates to finding a periodic evolution that would make an optimal cyclic cooling process.

The algorithm

Unlike the previous literature on HBAC, the operations in our algorithm do not depend on the state of the system. The entire procedure consists of repeating two steps:

1) Compression:
   Apply a fixed unitary to cool a part of the system while heating the rest. Our unitary has the form

$$U_{fix} = \begin{pmatrix} 1 & & & & \\ & \sigma_x & & & \\ & & \ddots & & \\ & & & \sigma_x & \\ & & & & 1 \end{pmatrix}$$

2) Reset :
   Reset the temperature of the rightmost qubit to the temperature of the heat-bath.

Analysis

We rigorously estimated the complexity of our algorithm. Since only the diagonal elements of the density matrix of the system determine the temperature, we formulate our algorithm in terms of probabilities over computational states. Our process can then be described in terms of a Markov chain with the transition matrix

$$T = \frac{1}{Z}\begin{pmatrix} e^{\epsilon} & e^{\epsilon} & 0 & \cdots & 0 \\ e^{-\epsilon} & 0 & e^{\epsilon} & \cdots & 0 \\ 0 & e^{-\epsilon} & 0 & \cdots & 0 \\ 0 & 0 & \cdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{-\epsilon} & e^{-\epsilon} \end{pmatrix}$$

where the polarization ϵ is inversely proportional to the temperature. We analytically compute the spectrum of T and find the stationary state and the mixing time.

Results

We present an HBAC technique whose operations do not require a perfect knowledge of the system and give a circuit for executing the compression step. The price we pay is an exponentially slower convergence compared with existing techniques. However, these existing techniques might require complicated unitaries to implement compression, which might mean that the complexity of these techniques might grow exponentially as well.

Open questions

Does independence on the state guarantee certain robustness? If so, can one derive a threshold for cooling?
Can one formulate oblivious versions of advanced HBAC methods?
Is it possible to perform oblivious cooling efficiently?

References

[1] Scalable NMR quantum computation, Schulman & Vazirani, 1998
[2] Physical limits of heat-bath algorithmic cooling, Schulman et al., PRL 2005
[3] Heat-Bath Algorithmic Cooling with optimal thermalization strategies, Alhambra et al., Quantum 2019
[4] Asymptotic bound for heat-bath algorithmic cooling, Raeisi & Mosca, PRL 2015
[5] Novel Technique for Robust Optimal Algorithmic Cooling, Raeisi et al., PRL 2019

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