The central idea of adiabatic quantum computing is to use a classically controlled time-dependent quantum system to prepare an eigenstate that encodes the answer to a computational problem. Although equivalent to the circuit model of computing, algorithm design and optimization can be difficult for adiabatic quantum computing. Here we address the question whether generalizing adiabatic quantum information protocols to use quantum, rather than classical control, can help address these issues. We find that coherent control and measurement provides a way to average different adiabatic evolutions in ways that cause their diabatic errors to cancel, allowing for adiabatic evolutions to combine the best characteristics of existing adiabatic optimizations strategies that are currently mutually exclusive. Finally, we show that this approach is polynomially equivalent to classically controlled adiabatic evolution; illustrating that this hybrid model is not unrealistic.

Coherently Controlled Quantum Adiabatic Evolutions

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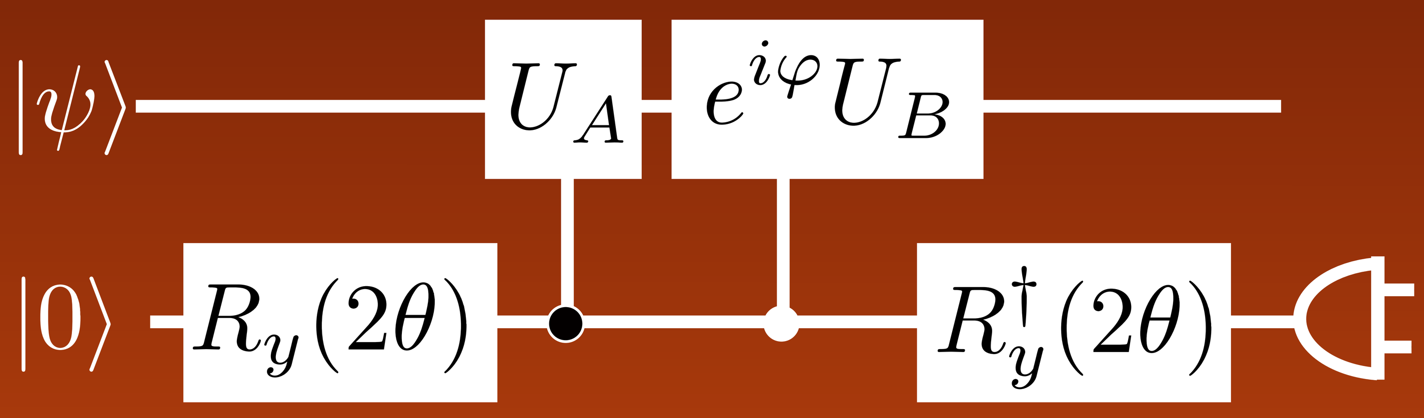
Linear combinations



Outcome 0 on ancilla yields a linear combination of unitaries

$$|\psi\rangle|0\rangle \rightarrow \left(\cos^2(\theta)e^{i\varphi}U_B + \sin^2(\theta)U_A\right)|\psi\rangle|0\rangle + \sin(\theta)\cos(\theta)\left(U_A - e^{i\varphi}U_B\right)|\psi\rangle|1\rangle$$

The success probability reaches

$$p(+) = 1 - O\left(\frac{1}{T}\right)$$

AQC

Slowly changing Hamiltonian

$$H(s) = f(s)H_1 + (1 - f(s))H_0$$

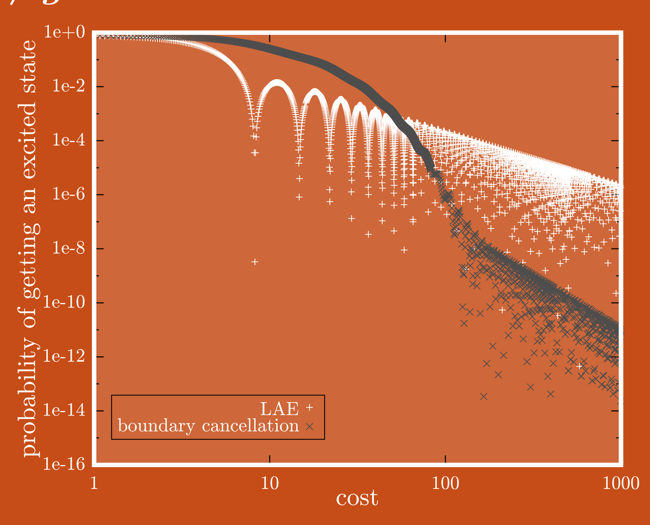
For slow enough evolution the system stays in its eigenstate

$$U(1,0)|g(0)\rangle = e^{-i\int_0^1 E_0(\xi)T d\xi}|g(1)\rangle + O\left(\frac{1}{T}\right)$$

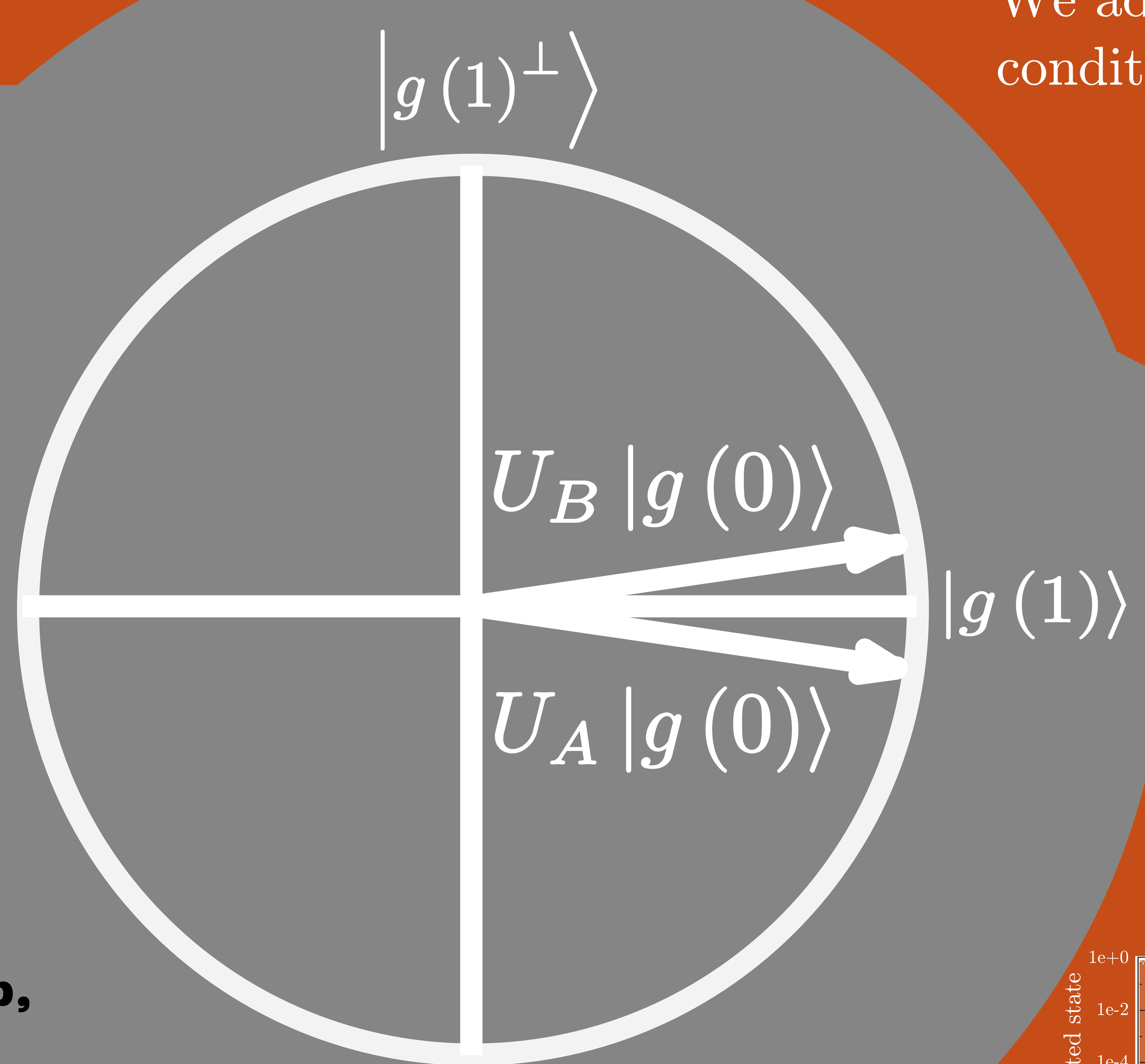
with error

$$\sum_{n\neq g} e^{-i\int_0^1 E_n(\xi)Td\xi} \left.\frac{\langle \dot{n}(s)|g(s)\rangle e^{i\int_0^s \gamma_{g,n}(\xi)d\xi T}}{-i\gamma_{g,n}(s)T}\right|_{s=0}^{1} |n(1)\rangle + O\left(\frac{1}{T^2}\right)$$

$$s = \frac{t}{T}$$
$$f(0) = 0$$
$$f(1) = 1$$



Local adiabatic evolution minimizes time to reach the adiabatic regime.
Boundary cancellation optimizes scaling in the adiabatic regime



Coherent control

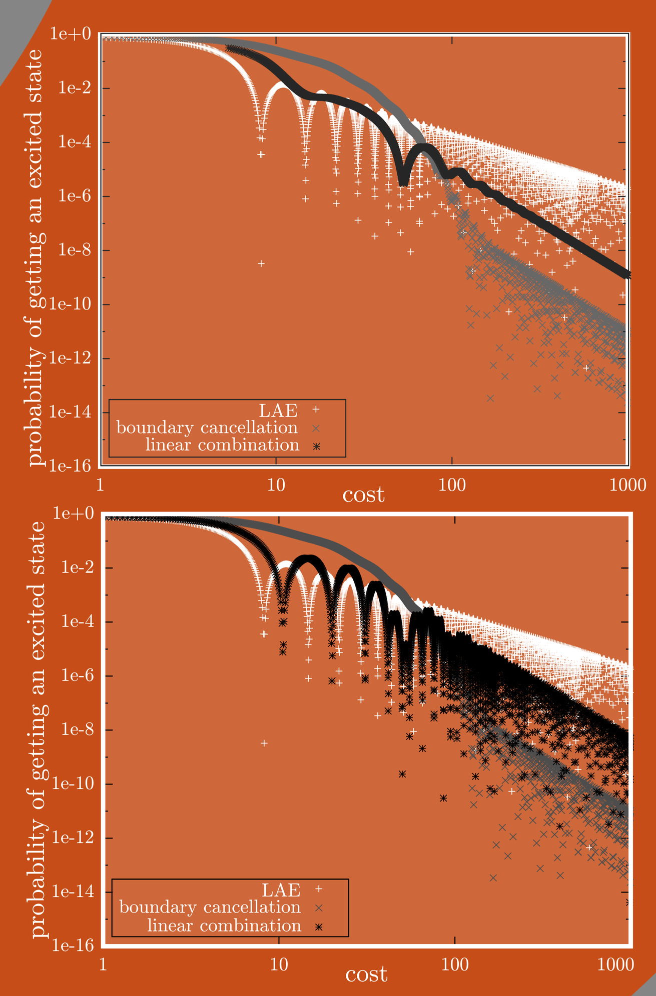
We add a small ancillary register and condition the adiabatic evolution on its state.

$$|k\rangle|\psi\rangle \rightarrow |k\rangle U_k|\psi\rangle$$

A measurement on ancillas allows us to implement a wider class of operations including linear combinations of unitaries.

2 levels

Convex combination of two evolutions with (almost) opposite errors



1) The evolutions have the same (signs of) derivatives at the end and opposite ones at the beginning.

Evolutions can be close to LAE - fast convergence.

2) The evolutions have opposite (signs of) derivatives at both boundaries.

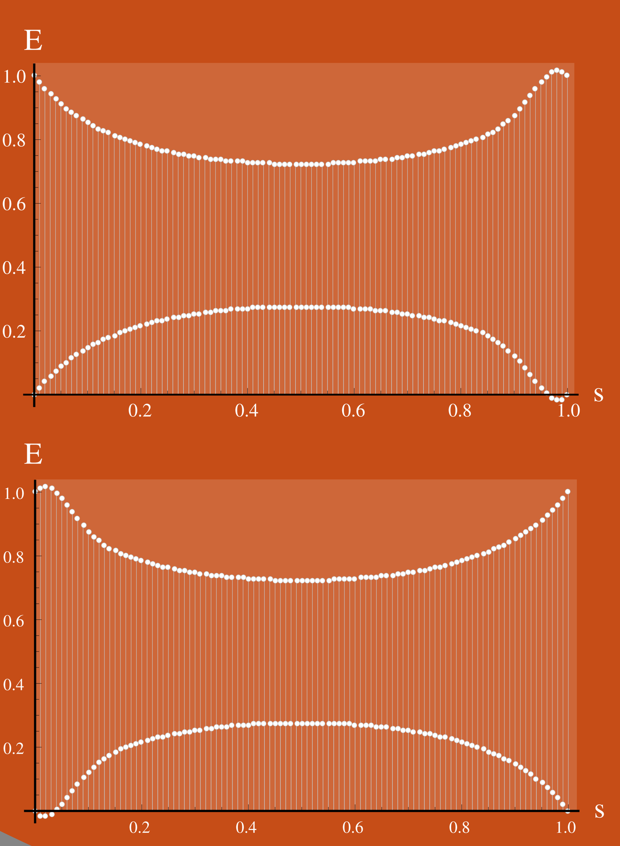
The cost is measured as

$$\int_0^T |H(t)|\,dt$$

N levels

1) Add more unitaries and solve a system of algebraic equations.

2) If H_1 and H_0 have the same spectra we can suppress all transitions at once by picking antisymmetric interpolations.



Specification of the model

Controlled adiabatic evolution is polynomially equivalent to the circuit model. The continuous evolution can be broken to discreet steps and simulated with quantum gates.

The Hamiltonian must be 3-times differentiable, row computable and sparse.

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+ our paper on the arxiv soon

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