On The Power Of Coherently Controlled Quantum Adiabatic Evolutions

Maria Kieferova

(RCQI, Slovak Academy of Sciences, Bratislava, Slovakia)

Nathan Wiebe

(QuArC, Microsoft Research, Redmond, WA, USA)

[arXiv:1403.6545]

Ground State

$$H = \begin{bmatrix} \cdots & & \\ & \ddots & \\ & & \cdots \end{bmatrix}$$

real eigenvalues = energies

ground state = the lowest energy

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real eigenvalues = energies

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Example:

$$H = -\sum_{j=0}^{N} Z_j \qquad \rightarrow \qquad |g\rangle = |00\ldots0\rangle$$

Evolution With A Hamiltonian

The Schrödinger equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\left|\psi(t)\right\rangle = -iH\left|\psi(t)\right\rangle$$

Eigenstates

$$H\left|n\right\rangle = E_n\left|n\right\rangle$$

$$\langle n|m\rangle = \delta_{m,n}$$

Evolution

$$\left|n(t)\right\rangle = e^{-iE_n t}\left|n(0)\right\rangle$$

Evolution With A *Time Dependent* Hamiltonian

The Schrödinger equation

$$\frac{\mathrm{d}}{\mathrm{d}t} |\psi(t)\rangle = -iH(t) |\psi(t)\rangle$$

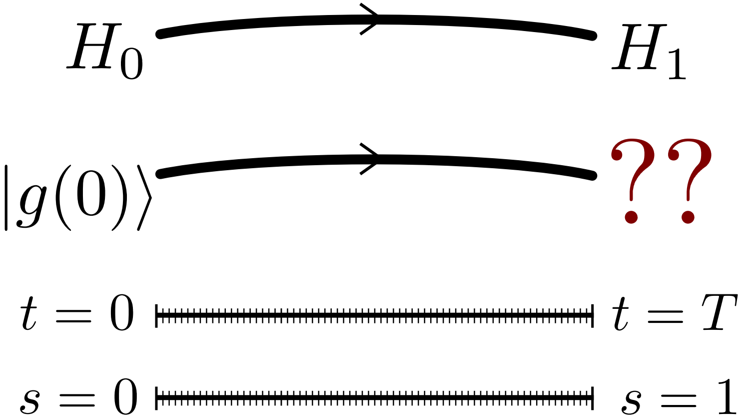
Instantaneous eigenstates

$$H(t) |n(t)\rangle = E_n(t) |n(t)\rangle$$

$$\langle n(t)|m(t)\rangle = \delta_{m,n}$$

Evolution

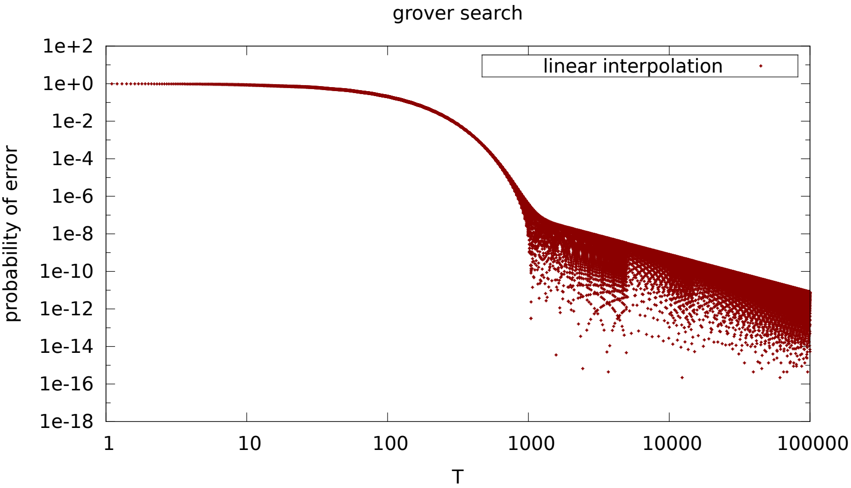
$$|n(t)\rangle \neq e^{-iE_n t} |n(0)\rangle$$


H_0
H_1
$|g(0)\rangle$
??
$t = 0$
$t = T$
$s = 0$
$s = 1$

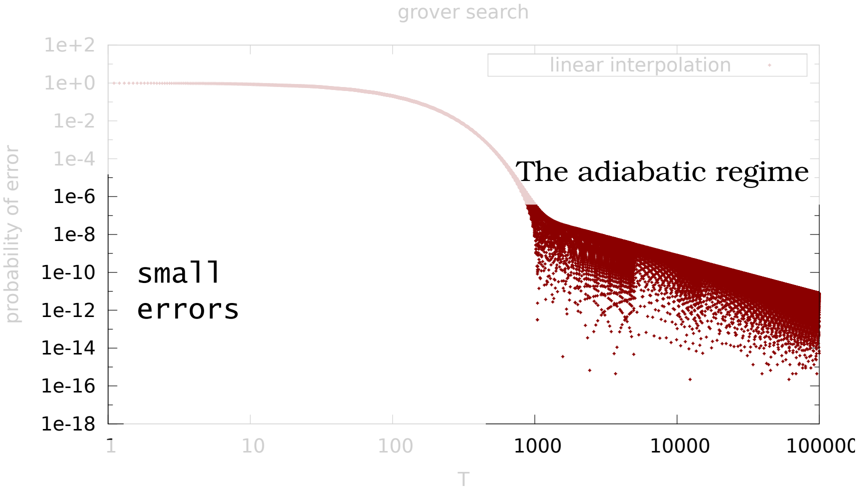
goal:

stay in the ground state

Probability Of Error

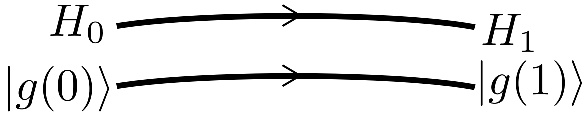


Probability Of Error



Adiabatic Computation

- H_0 has a ground state which is easy to prepare
- H_1 has an interesting ground state



- Complexity [Farhi, E. et al., (2000)]
- Universal [Aharonov, D. et al.,(2004)]
- Quantum simulation, state preparation

The Adiabatic Theorem

The error is small when

$$T \gg \frac{1}{\min \gamma_{e1,g}^2(s)}$$

for bounded $||H||$ and its derivatives

The Adiabatic Theorem

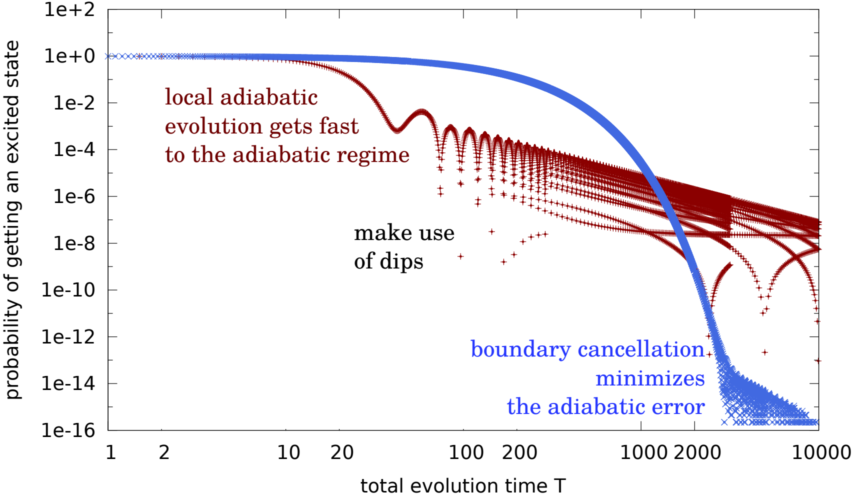
Let $H : [0,1] \to \mathbb{C}^{N\times N}$ be a Hamiltonian that is differentiable three times and has a minimum eigenvalue gap between any two non-degenerate eigenvectors of $\gamma_{min} > 0$. If we take $|g(s)\rangle$ to be a non-degenerate instantaneous eigenstate of $H(s)$ and $\Delta_1 \in o(\gamma_{min} T^2)$, then the **error** in the adiabatic approximation, $(1 - |g(1)\rangle\langle g(1)|)\, U(1,0)\, |g(0)\rangle$, is

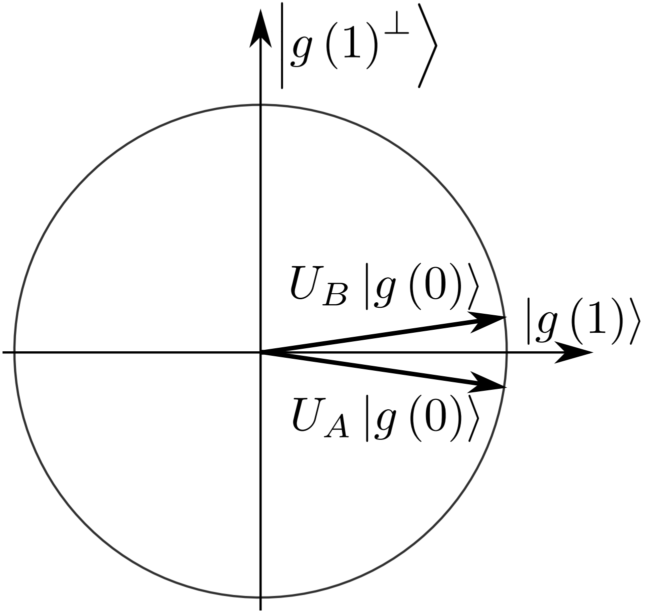
$$\sum_{n\neq g} e^{-i\Phi_n} \left. \frac{\langle \dot{n}(s)|g(s)\rangle e^{-i\int_0^s \gamma_{g,n}(\xi)d\xi T}}{\gamma_{g,n(s)} T}\right|_{s=0}^{1} |n(0)\rangle + O\left(\frac{\Delta_1}{\gamma_{min} T^2}\right)$$

where $\Phi_n = \int_0^1 E_N(\chi)d\chi T$, $\gamma_{g,n} = E_g(s) - E_n(s)$, $\Delta_1 = \frac{1}{\gamma_{min}}\left(\frac{||\ddot{H}||}{\gamma_{min}} + \frac{||\dot{H}||^3}{\gamma_{min}^3} + \frac{||\dot{H}||^6}{\gamma_{min}^6}\right)$ and the phase of each instantaneous eigenstate is chosen such that $\langle \dot{n}(s)|n(s)\rangle = 0$ for every $s \in [0,1]$ and every n.

[Cheung, Hoyer, Wiebe (2011)]

Error minimization methods





$\left|g(1)^{\perp}\right\rangle$
$U_B\,|g(0)\rangle$
$|g(1)\rangle$
$U_A\,|g(0)\rangle$

Coherently Controlled Adiabatic Evolution

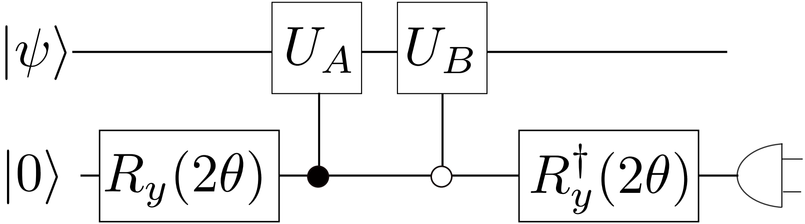
$$U_k : \; H(f_k(s)) = f_k(s)H_1 + (1 - f_k(s))H_0$$

$$|\psi\rangle \, |k\rangle \to U_k \, |\psi\rangle \, |k\rangle$$

measurement and post-selection allow us to implement a wider class of operations

[I. Hen, (2014)], [P. Zanardi and M. Rasetti, (1999)]

Linear Combinations



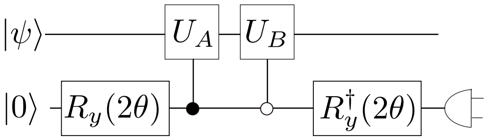
$$
\begin{aligned}
|\psi\rangle|0\rangle &\to |\psi\rangle(\cos\theta|0\rangle + \sin\theta|1\rangle) \\
&\to \cos\theta|\psi\rangle|0\rangle + \sin\theta U_A|\psi\rangle|1\rangle \\
&\to \cos\theta U_B|\psi\rangle|0\rangle + \sin\theta U_A|\psi\rangle|1\rangle \\
&\to \left(\cos^2\theta U_B + \sin^2\theta U_A\right)|\psi\rangle|0\rangle \\
&+ \sin\theta\cos\theta\,(U_A - U_B)|\psi\rangle|1\rangle\,.
\end{aligned}
$$

[Wiebe, N. and Childs, A.M., 2012.]

- apply U_A, U_B in parallel

Success Probability

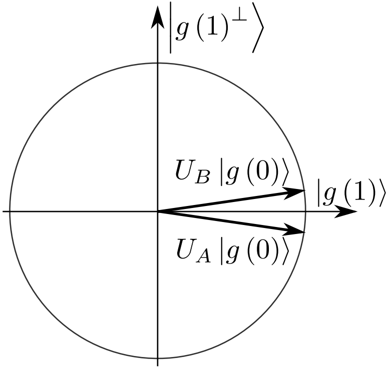
$$U_A: \; H(f(s)) = f(s)H_1 + (1-f(s))H_0$$
$$U_B: \; H(g(s)) = g(s)H_1 + (1-g(s))H_0$$



Success probability$= 1 - \|(U_A - U_B)\,|\psi\rangle\|^2$
Ground states must pick similar phases

$$U \to e^{+i\int_0^1 E_0(s)dsT}U$$

Error



$$0 = \cos^2\theta \sum_{n\neq g}\left[\frac{\langle \dot{n}(1)|g(1)\rangle}{\gamma_{g,n(1)}T} - \frac{\langle \dot{n}(0)|g(0)\rangle e^{i\int_0^1 \gamma_{g,n}(\xi)d\xi T}}{\gamma_{g,n(0)}T}\right]|n(1)\rangle$$

$$+ \sin^2\theta \sum_{n\neq g}\left[\frac{\langle \dot{n}'(1)|g(1)\rangle}{\gamma_{g,n(1)}T'} - \frac{\langle \dot{n}'(0)|g(0)\rangle e^{i\int_0^1 \gamma'_{g,n}(\xi)d\xi T'}}{\gamma_{g,n(0)}T'}\right]|n(1)\rangle$$

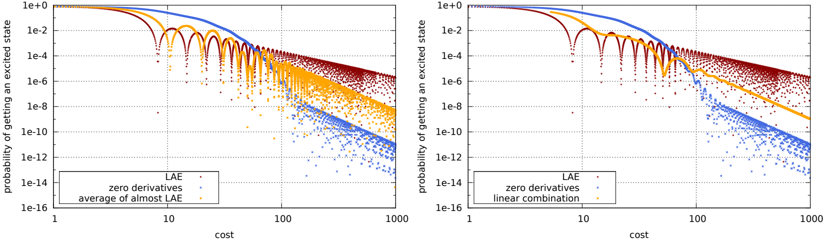
One excited state

$$H(f(s)) = f(s)H_1 + (1 - f(s))H_0$$
$$H(g(s)) = g(s)H_1 + (1 - g(s))H_0$$

- f, g generate evolutions with opposite errors
- derivatives of the Hamiltonians at the end must be opposite
- freedom for the initial boundary

$$0 = \cos^2\theta \left[\frac{\langle \dot{n}(1)|g(1)\rangle}{\gamma_{g,n(1)}T} - \frac{\langle \dot{n}(0)|g(0)\rangle e^{i\int_0^1 \gamma_{g,n}(\xi)d\xi T}}{\gamma_{g,n(0)}T} \right] |n(1)\rangle$$
$$+ \sin^2\theta \left[\frac{\langle \dot{n}'(1)|g(1)\rangle}{\gamma_{g,n(1)}T'} - \frac{\langle \dot{n}'(0)|g(0)\rangle e^{i\int_0^1 \gamma'_{g,n}(\xi)d\xi T'}}{\gamma_{g,n(0)}T'} \right] |n(1)\rangle$$

One excited state



- gets fast to the adiabatic regime
- favorable error scaling in the AR

More transitions

1. Linear combinations from more evolutions
    - 3 levels and 4 evolutions
    - equations for higher dimensional system

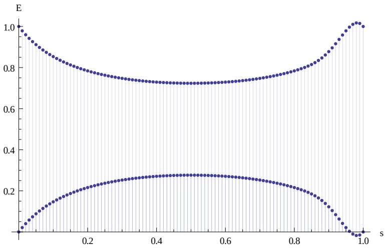
More transitions

1. Linear combinations from more evolutions
   - 3 levels and 4 evolutions
   - equations for higher dimensional system
2. Exploit symmetry

Symmetric evolutions

H_0 and H_1 have the same spectrum

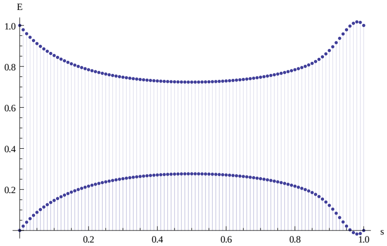
$$H(0.5 - s) = H'(0.5 + s)$$
$$\dot{f}(s)\Big|_{s=0} = -\dot{f}(s)\Big|_{s=1}$$

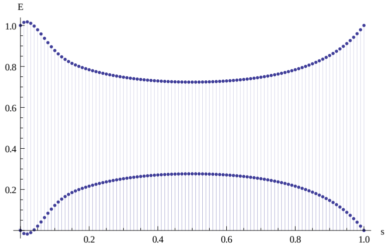


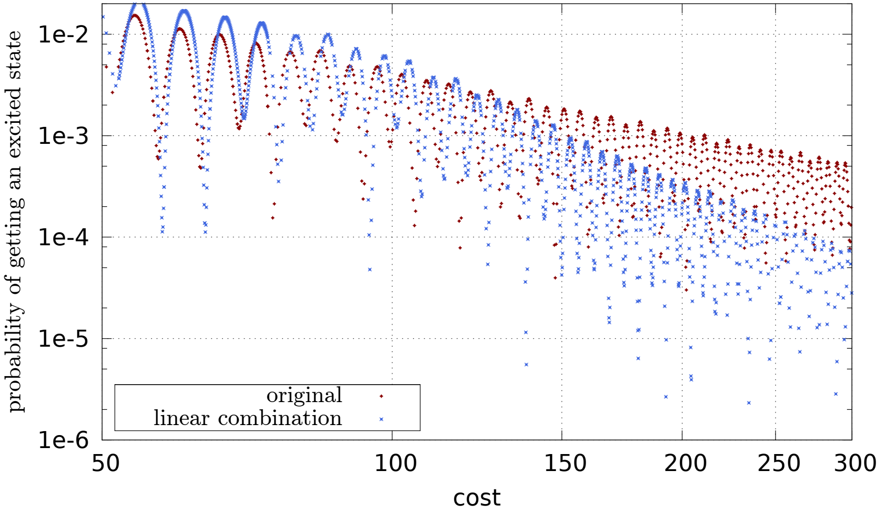
Symmetric evolutions

$$0 = \cos^2\theta \sum_{n\neq g} \left[\frac{\langle \dot{n}(1)|g(1)\rangle}{\gamma_{g,n(1)}T} - \frac{\langle \dot{n}(0)|g(0)\rangle e^{i\int_0^1 \gamma_{g,n}(\xi)d\xi T}}{\gamma_{g,n(0)}T} \right] |n\,(1)\rangle$$

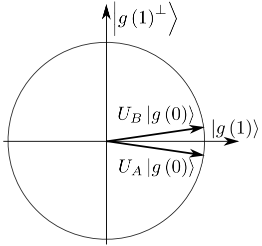
$$+ \sin^2\theta \sum_{n\neq g} \left[\frac{\langle \dot{n}'(1)|g(1)\rangle}{\gamma_{g,n(1)}T'} - \frac{\langle \dot{n}'(0)|g(0)\rangle e^{i\int_0^1 \gamma'_{g,n}(\xi)d\xi T'}}{\gamma_{g,n(0)}T'} \right] |n\,(1)\rangle$$



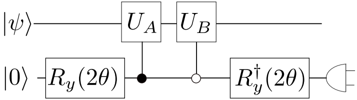



probability of getting an excited state
1e-2
1e-3
1e-4
1e-5
1e-6
50
100
150
200
250
300
cost
original
linear combination

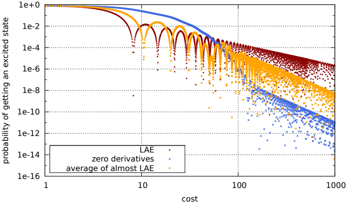
MIND THE GAP



$|g(1)^{\perp}\rangle$
$U_B|g(0)\rangle$
$|g(1)\rangle$
$U_A|g(0)\rangle$

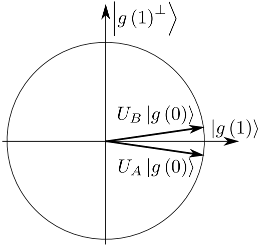


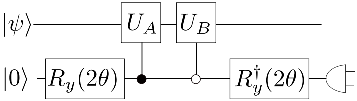
$|\psi\rangle$
$|0\rangle$
U_A
U_B
$R_y(2\theta)$
$R_y^{\dagger}(2\theta)$

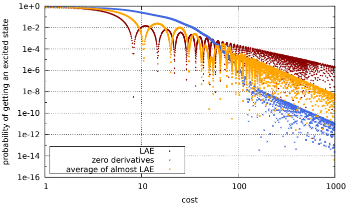


probability of getting an excited state
cost
1e+0
1e-2
1e-4
1e-6
1e-8
1e-10
1e-12
1e-14
1e-16
1
10
100
1000
LAE
zero derivatives
average of almost LAE

MIND THE GAP







Thanks for your attention!