Oblivious Algorithmic Cooling

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based on

Novel Technique for Robust Optimal Algorithmic Cooling,

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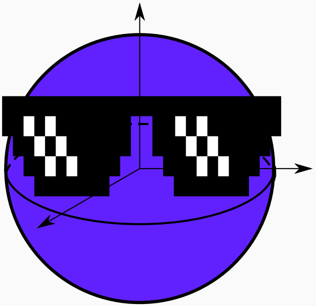
PRL 2019

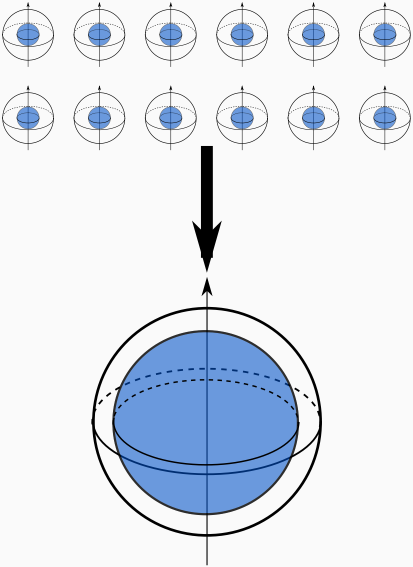
Overview

- Cooling in a closed system
- Heat bath algorithmic cooling
- Oblivious implementation



Algorithmic Cooling

Can We Extract One Good Qubit Out Of Many Dirty Ones?



[Schulman, Vazirani (1998)]

Framework

Obtain a single purified qubit using *n* qubits and logical operations.

Each qubit is in a thermal state

$$\rho_{therm} = \frac{1}{e^{\epsilon} + e^{-\epsilon}} \begin{pmatrix} e^{\epsilon} & 0 \\ 0 & e^{-\epsilon} \end{pmatrix}$$

where ϵ is the “polarization” ($\propto \frac{1}{T}$, typically very small).

”Cooling” refers to approximately extracting a qubit in state $|0\rangle\langle 0|$.

”Algorithmic” means that we are purifying a subsystem by applying logical operations.

Assume that the density matrices of our states are always diagonal.

Result

New method for algorithmic cooling:

- uses heat bath and converges to the optimal state
- repeated application of the same operation
- easier implementation, more robustness
- slow convergence in terms of the number of qubits

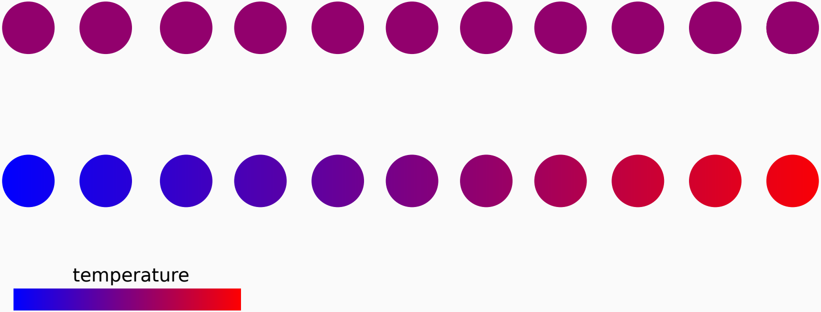
[Raeisi et al. 2019]

Fundamental Questions

- What are the physical limits of algorithmic cooling?
- What are the required resources?
- How do the assumptions about control affect the points above?

Closed System Cooling

Apply unitaries on the system of qubits.



Cooling parts of the system heats up the rest.

Example: 3 Qubits

$$\rho = \frac{1}{Z}\begin{pmatrix} e^{3\epsilon} & . & . & . & . & . & . & . \\ . & e^{\epsilon} & . & . & . & . & . & . \\ . & . & e^{\epsilon} & . & . & . & . & . \\ . & . & . & e^{-\epsilon} & . & . & . & . \\ . & . & . & . & e^{\epsilon} & . & . & . \\ . & . & . & . & . & e^{-\epsilon} & . & . \\ . & . & . & . & . & . & e^{-\epsilon} & . \\ . & . & . & . & . & . & . & e^{-3\epsilon} \end{pmatrix}$$

Observe that $\rho_{n,n} = \frac{1}{Z}e^{n-2|n|}$ where $|n|$ is the Hamming weight of n in binary.

Compression

“Carry out a permutation of the computation basis states $x \in \{0,1\}^n$ such that states with low Hamming weight should be reordered with a long prefix of 0’s.”

[Schulman, Vazirani (1998)]

Example: 3 Qubits

Apply a unitary to order the diagonal elements

$$\begin{pmatrix} e^{3\epsilon} & . & . & . & . & . & . & . \\ . & e^{\epsilon} & . & . & . & . & . & . \\ . & . & e^{\epsilon} & . & . & . & . & . \\ . & . & . & e^{-\epsilon} & . & . & . & . \\ . & . & . & . & e^{\epsilon} & . & . & . \\ . & . & . & . & . & e^{-\epsilon} & . & . \\ . & . & . & . & . & . & e^{-\epsilon} & . \\ . & . & . & . & . & . & . & e^{-3\epsilon} \end{pmatrix} \rightarrow \begin{pmatrix} e^{3\epsilon} & . & . & . & . & . & . & . \\ . & e^{\epsilon} & . & . & . & . & . & . \\ . & . & e^{\epsilon} & . & . & . & . & . \\ . & . & . & e^{\epsilon} & . & . & . & . \\ . & . & . & . & e^{-\epsilon} & . & . & . \\ . & . & . & . & . & e^{-\epsilon} & . & . \\ . & . & . & . & . & . & e^{-\epsilon} & . \\ . & . & . & . & . & . & . & e^{-3\epsilon} \end{pmatrix}$$

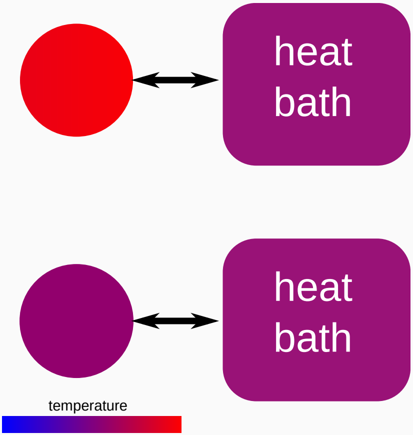
Compression is limited by Shannon bound.

Heat Bath Algorithmic Cooling (HBAC)

Add a Heat Bath

Allow parts of the system (the "reset" qubit) to interact with the heat bath

$O_{reset}(\rho) = Tr_{reset}[\rho] \otimes \rho_{therm}$



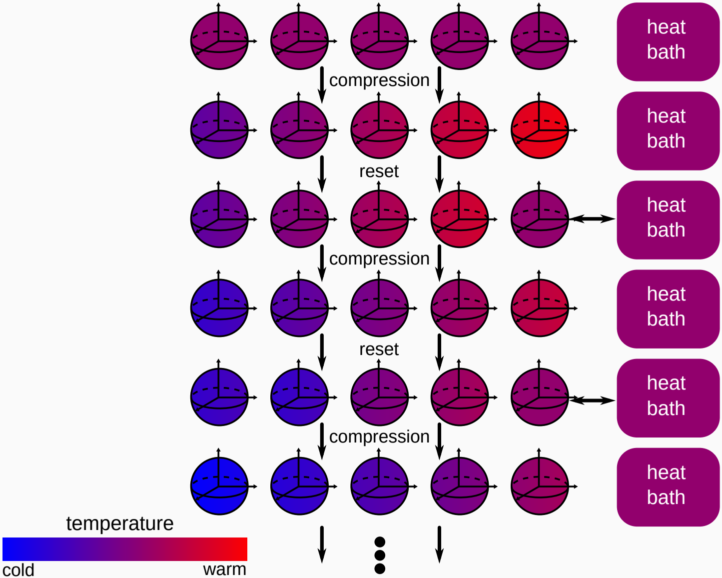
[P. O. Boykin et al. (2002), Schulman et al. (2005)]

Example: 3 Qubits

For a state with $p_1, p_2, \ldots, p_8$ on the diagonal

$$\frac{1}{Z}\begin{pmatrix} (p_1+p_2)e^{\epsilon} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & (p_1+p_2)e^{-\epsilon} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & (p_3+p_4)e^{\epsilon} & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & (p_3+p_4)e^{-\epsilon} & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & (p_5+p_6)e^{\epsilon} & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & (p_5+p_6)e^{-\epsilon} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & (p_7+p_8)e^{\epsilon} & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & (p_7+p_8)e^{-\epsilon} \end{pmatrix}$$

The Partner Pairing Algorithm (PPA)



Two Steps

Compression

Apply a unitary to re-order the diagonal elements.

Reset

Reset the hottest qubit to the original polarization.

Simple scenario: only one qubit interacts with the heat bath (reset qubit), focus on the final polarization of only one qubit (target)

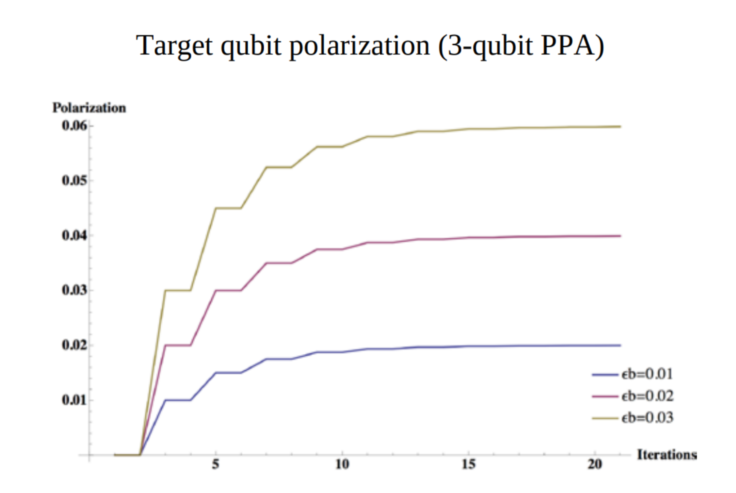
The Limitation of HBAC

There exists a maximum polarization $\epsilon_f < \infty$ specified the number of qubits *n* and the heat bath polarization for HBAC schemes (with above assumptions).

PPA achieves the maximum polarization in *poly*(n) steps.

[Raeisi, Mosca (2014)]

Example: 3 Qubits



[Park et al. (2016)]

Two Big Problems

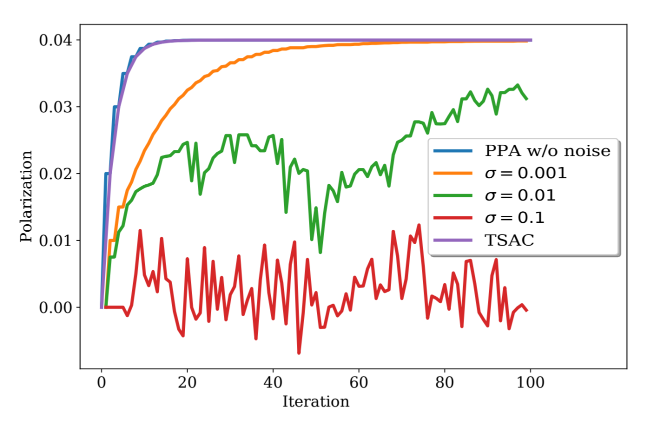
- What is the complexity of implementing the compression unitaries?

  How do we implement the compression as circuits?

- We require perfect knowledge of the state at all times to apply the correct operations.

  Do we need to perform tomography to know the states?

Using the Correct Unitary is Important



Using a unitary corresponding to a slightly different state does not lead to cooling

Oblivious Implementation

Our Contribution

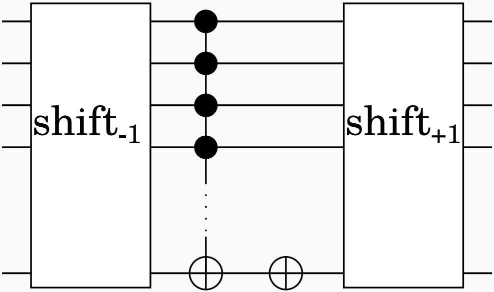
Apply a fixed, state independent unitary in each round.

- this unitary can be implemented with a polynomially-sized circuit
- no need for tomography
- some inherent robustness (conjectured)

The Compression Unitary

$$U_{fix} = \begin{pmatrix} 1 & & & & \\ & \sigma_x & & & \\ & & \ddots & & \\ & & & \sigma_x & \\ & & & & 1 \end{pmatrix}$$

requires $\mathcal{O}(n^2)$ elementary gates.



Cooling as a Markov Chain

Repeated application of the channel

$$U_{fix}^{\dagger}\left(Tr_{reset}[\rho]\otimes\rho_{therm}\right)U_{fix}$$

converges to the asymptotic state of PPA.

We can analyze this map as a Markov chain on the probability vector diag(ρ) with a transfer matrix

$$T=\frac{1}{Z}\begin{pmatrix} e^{\epsilon} & e^{\epsilon} & 0 & \cdots & 0 \\ e^{-\epsilon} & 0 & e^{\epsilon} & \cdots & 0 \\ 0 & e^{-\epsilon} & 0 & \cdots & 0 \\ 0 & 0 & \cdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{-\epsilon} & e^{-\epsilon} \end{pmatrix}.$$

Spectrum of T

T is (mostly) translationary invariant and stochastic

$$T = \frac{1}{Z}\begin{pmatrix} e^{\epsilon} & e^{\epsilon} & 0 & \cdots & 0 \\ e^{-\epsilon} & 0 & e^{\epsilon} & \cdots & 0 \\ 0 & e^{-\epsilon} & 0 & \cdots & 0 \\ 0 & 0 & \cdots & \ddots & \vdots \\ 0 & 0 & \cdots & e^{-\epsilon} & e^{-\epsilon} \end{pmatrix}.$$

We analytically compute its spectrum using a wave-like ansatz.

Unique +1 eigenvector is the asymptotic state

$$\text{diag}(\rho) \propto (1, e^{-2\epsilon}, e^{-4\epsilon}, \dots) \otimes \rho_{therm}.$$

For $\epsilon \ll \frac{1}{2^n}$, the polarization of the first qubit is $\approx 2^{n-2}\epsilon$.

Complexity

The number of rounds of cooling is determined by the gap of T.

– Number of rounds: $\mathcal{O}(2^n)$ (polynomial for PPA)

– Total complexity: $\mathcal{O}(n^2 2^n)$

+ There are strategies that can improve the scaling.

+ Number of qubits is not a fixed parameter.

Open Questions

- Robustness of cooling

  Can we handle imperfect gates? What is the noise threshold?
- More efficient cooling algorithms

  Could we get a $poly(n)$ algorithm?
- Computational complexity of different cooling paradigms

  A lower cooling limit is possible for more complex interaction between the qubits and the heat-bath. What is the complexity?

  Can we cool without perfect knowledge about the state?

Conclusion

- New heat bath algorithmic cooling method that converges to the optimal state.
- Fixed operation in every round, no need to know the state, easier to implement.
- Not efficient for a large number of qubits.

Thank you!